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Abstract

We present a theoretical model in which children transfer income to their parent non-cooperatively among their siblings. In this gift economy, we observe that fertility declines over time, capital accumulation can be non-monotonic, and the steady-state equilibrium tends to be dynamically efficient.

JEL Classification: J13, J14, O41

Keywords: Gift economy, Dynamic efficiency, Fertility, Sibling rivalry

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1 Introduction

Intergenerational transfers within a family are one of the most controversial issues in public economics and population economics because they may neutralize public policies related to income redistribution (Barro (1974)), and because they may be a source of demographic transition (Galor and Weil (2000)). While descending transfers from parents to children, such as bequests, inter-vivos transfers, and education, have been analyzed extensively, research on ascending transfers from children to parents, such as gifts, attention, and informal care, have not been well developed, especially in the dynamic context. One reason is that the gift economy is dynamically inefficient (Carmichael (1982)), which is a principle rejected in most developed countries (Abel et al. (1989))[1].

Some of the exceptions include O'Connell and Zeldes (1993) and Wigger (2001). O'Connell and Zeldes (1993) show that the gift economy is dynamically efficient if parents act as leaders, that is, if parents choose savings by using a linear gift function that represents their children's optimal response to the savings choice. Perceiving their children as altruistic, parents have an incentive to save less in order to receive more gifts. This induced undersaving makes the economy dynamically efficient. In the Romer (1986) endogenous growth model, Wigger (2001) shows that the gift economy is dynamically efficient in the sense that the social return of capital is larger than the equilibrium growth rate.

The purpose of the paper is to give another rationale for the gift economy to be dynamically efficient. The focal points are endogenous fertility and sibling rivalry[2]. Parents choose the number of children they bear, taking gifts transferred from children as given. Each child chooses a gift to give to his parents, taking the amount of gifts his siblings choose as given. In this scenario, an interaction among the rates of fertility, gift-giving, and saving arises. Suppose that the fertility rate is low in equilibrium. Then the gift rate would be high because the decreased size of the family alleviates the free-rider problem within the family. The saving rate would be low because parents rely on gifts from children in retirement. Consequently, the low saving rate could make the economy dynamically efficient. In a simple overlapping generations model, we show that this scenario could be realized under fairly weak conditions. In addition to the dynamic efficiency condition at a steady state, we also show that the fertility rate declines over time and that capital accumulation could be non-monotonic in the transition process. Intuitively, the fertility dynamics stems from a positive relationship between the parents' fertility decision and the children's fertility decision. The non-monotonicity stems from a difference in the adjustment speed of capital and fertility.

The paper is organized as follows. In section 2, we introduce a basic model. In section 3, we analyze the equilibrium dynamics of the rates of fertility, gift-giving, and saving. In section 4, we derive a dynamic efficiency condition when siblings are non-cooperative. The final section concludes the paper.

[1] The problem is also tackled in the two-sided altruism model (Abel (1987), Kimball (1987), Laitner (1988), and Blackburn and Cipriani (2005) among others).

[2] In a static model, Chang and Weisman (2005) show that sibling rivalry makes parental transfers inefficient.

2 The model

We use a two-period overlapping generations model with endogenous fertility. In each period, identical individuals are newly born into the economy and live for two periods. In the first period, they supply one unit of labor and allocate their income among consumption, saving, child-rearing, and a gift to their parents. In the second period, they retire from business and receive capital income and gifts from their children to consume. In each period, identical firms produce a good by employing capital and labor. Markets are competitive, and the economy is closed.

We refer to the group of individuals that are born at period t as generation t. Denoting the population of generation t by N_t, and the number of children each individual in generation t has by n_t, we have

$$\frac{N_{t+1}}{N_t} = n_t \tag{1}$$

The utility function of an individual in generation t is given by

$$U_t = u_t + \theta u_{t-1}$$

where

$$u_t = u(c_{1t}, c_{2t+1}, n_t) = \ln c_{1t} + \beta \ln c_{2t+1} + \eta \ln n_t$$

u_t stands for an individual's own lifetime utility, which consists of young-age consumption c_{1t}, old-age consumption c_{2t+1}, and the number of children n_t. $\beta > 0$ is a private discount factor, and $\eta > 0$ is a preference parameter attached to the number of children. Besides his own lifetime utility, this individual cares about his parent's utility, $u_{t-1} = u(c_{1t-1}, c_{2t}, n_{t-1})$, which is weighted by an altruistic parameter, $\theta > 0$. This altruism causes children to transfer income to their parent.

His budget constraints in the first and second period, respectively, are given by

$$(1 - g_t - s_t - \phi n_t)w_t = c_{1t} \tag{2}$$

$$(1 + r_{t+1})s_t w_t + n_t g_{t+1} w_{t+1} = c_{2t+1} \tag{3}$$

where g_t and s_t stand for a gift rate and a saving rate, respectively. w_t is a wage rate in period t, and r_{t+1} is an interest rate in period $t+1$. ϕw_t is a child-rearing cost per child.

In addition, this individual is interested in his parent's old-age consumption, c_{2t}. Because he or she has $(n_{t-1} - 1)$ siblings, he would expect his parent's old-age consumption to be

$$c_{2t} = (1 + r_t)s_{t-1}w_{t-1} + \sigma n_{t-1} g_t w_t + (1 - \sigma)[g_t w_t + (n_{t-1} - 1)\bar{g}_t w_t] \tag{4}$$

where σ is a binary parameter. If $\sigma = 1$, then $c_{2t} = (1+r_t)s_{t-1}w_{t-1} + n_{t-1}g_t w_t$. He expects his siblings to choose the same amount of gift as he chooses. We refer to this case as siblings who are 'cooperative'. If $\sigma = 0$, then $c_{2t} = (1 + r_t)s_{t-1}w_{t-1} + g_t w_t + (n_{t-1} - 1)\bar{g}_t w_t$. He chooses the gift g_t, taking his siblings' gift $\bar{g}_t$ as given. We refer to this case as siblings who are 'non-cooperative'.

From equations (2) and (3), the lifetime budget constraint is given by

$$(1 - g_t - \phi n_t)w_t + \frac{n_t g_{t+1} w_{t+1}}{1 + r_{t+1}} = c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} \tag{5}$$

The optimization problem is to choose c_{1t}, c_{2t+1}, n_t, and g_t to maximize utility subject to equations (5) and (4), while taking n_{t-1}, s_{t-1}, g_{t+1}, and $\bar{g}_t$ as given.

The first-order conditions for c_{1t}, c_{2t+1}, n_t, and g_t require

$$\begin{aligned}
\frac{1}{c_{1t}} - \mu_t &= 0 \\
\frac{\beta}{c_{2t+1}} - \frac{\mu_t}{1 + r_{t+1}} &= 0 \\
\frac{\eta}{n_t} - \mu_t \phi w_t + \frac{\mu_t g_{t+1} w_{t+1}}{1 + r_{t+1}} &= 0 \\
\frac{\beta\theta[\sigma n_{t-1} w_t + (1-\sigma) w_t]}{c_{2t}} - \mu_t w_t &\leq 0 \quad \text{with } = \text{ if } g_t > 0
\end{aligned}$$

where μ_t stands for a multiplier attached to equation (5).

Assuming that the gift is operative, we have, in a symmetric equilibrium,

$$s_t + \phi n_t = \frac{\beta + \eta}{1 + \beta + \eta}(1 - g_t) \tag{6}$$

$$n_t \left[\phi - \frac{g_{t+1} w_{t+1}}{(1 + r_{t+1}) w_t}\right] = \frac{\eta}{1 + \beta + \eta}(1 - g_t) \tag{7}$$

$$\frac{(1 + r_t) s_{t-1} w_{t-1}}{w_t} + n_{t-1} g_t = \frac{\beta\theta}{1 + \beta + \eta}(1 - \sigma + \sigma n_{t-1})(1 - g_t) \tag{8}$$

Equation (6) implies that the propensity to consume based on the after-transfer income $(1 - g_t)w_t$ is constant and given by $(1 + \beta + \eta)^{-1}$. Equation (7) implies that the net marginal cost of having children is equal to the marginal benefit at the optimum. Equation (8) implies that the marginal cost of gift transfers is equal to the marginal benefit at the optimum.

The production technology is represented by a constant-returns-to-scale production function,

$$Y_t = F(K_t, L_t)$$

Assuming that factor markets are competitive and that capital fully depreciates in one period, we have

$$\begin{aligned}
1 + r_t &= f'(k_t) \\
w_t &= f(k_t) - k_t f'(k_t)
\end{aligned}$$

where $k_t = K_t/L_t$ is a capital-labor ratio, and $f(k_t) = F(k_t, 1)$ is per capita output. For tractability, we assume:

Assumption 1: The income share of capital is constant over time:

$$\frac{k_t f'(k_t)}{f(k_t)} = \alpha \tag{9}$$

Market clearing conditions for labor, capital, and good are given respectively by

$$\begin{aligned} N_t &= L_t \\ K_{t+1} &= N_t s_t w_t \\ Y_t &= N_t c_{1t} + N_{t-1} c_{2t} + N_{t+1} \phi w_t + K_{t+1} \end{aligned}$$

Because the model is closed, the goods market clearing condition can be given by Walras' law. From the capital market clearing condition, we have

$$k_{t+1} = \frac{s_t w_t}{n_t} \tag{10}$$

With equations (9) and (10), equations (7) and (8) become

$$\phi n_t = \frac{1-\alpha}{\alpha} g_{t+1} s_t + \frac{\eta}{1+\beta+\eta}(1-g_t) \tag{11}$$

$$\left(\frac{\alpha}{1-\alpha} + g_t\right) n_{t-1} = \frac{\beta\theta}{1+\beta+\eta}(1-\sigma+\sigma n_{t-1})(1-g_t) \tag{12}$$

Equations (6), (11), and (12) determine the law of motion of g_t, s_t, and n_t.

3 Equilibrium and dynamics

3.1 Cooperative siblings

In this section, we analyze a benchmark case of $\sigma = 1$, that is, each child expects his siblings choose the same amount of gift that he chooses.

The equilibrium is specified by equations (6), (11), and

$$\frac{\alpha}{1-\alpha} + g_t = \frac{\beta\theta}{1+\beta+\eta}(1-g_t) \tag{13}$$

Equation (13) implies that the gift rate is constant over time. Equations (6) and (11) imply that the saving rate and the fertility rate are also constant over time. Specifically, we have the following proposition.

Proposition 1 *In the cooperative siblings model, the rate of fertility, gift-giving, and saving are constant over time and given respectively by*

$$n_t = n^C \equiv \frac{(\beta+\eta)\theta - \alpha(1+\beta+\eta+\beta\theta)}{\phi\theta(1-\alpha)(1+\beta+\eta+\beta\theta)} \tag{14}$$

$$g_t = g^C \equiv \frac{\beta\theta - \alpha(1+\beta+\eta+\beta\theta)}{(1-\alpha)(1+\beta+\eta+\beta\theta)} \tag{15}$$

$$s_t = s^C \equiv \frac{\alpha}{(1-\alpha)\theta} \tag{16}$$

The interior solutions require

$$\theta > \frac{\alpha(1+\beta+\eta)}{(1-\alpha)\beta}$$

The process of capital accumulation is straightforward. Substituting equations (14) and (16) into equation (10), we have

$$k_{t+1} = \frac{\phi\alpha(1-\alpha)(1+\beta+\eta+\beta\theta)}{(\beta+\eta)\theta - \alpha(1+\beta+\eta+\beta\theta)} f(k_t) \tag{17}$$

Given an initial condition, k_0, the capital-labor ratio converges monotonically to a unique steady state. Figure 1 illustrates the relationship between fertility and capital accumulation in the cooperative siblings model.

[Figure 1 is here]

3.2 Non-cooperative siblings

In this section, we analyze a case of $\sigma = 0$, that is, each child chooses his gift while taking his siblings' gifts as given.

The equilibrium is specified by equations (6), (11), and

$$\left(\frac{\alpha}{1-\alpha} + g_t\right) n_{t-1} = \frac{\beta\theta}{1+\beta+\eta}(1-g_t) \tag{18}$$

In contrast to the cooperative siblings model, the gift rate depends on the number of siblings. This is due to the free-rider problem within the family. Each child has an incentive to decrease transfers to his parent if he has many siblings.

Equation (18) specifies a gift function $g_t = g(n_{t-1})$. Substituting this function into equations (6) and (11), we have $n_t = n(n_{t-1})$ and $s_t = s(n_{t-1})$. Specifically, we have the following proposition.

Proposition 2 *In the non-cooperative siblings model, the law of motion of the fertility rate is given by*

$$n_t = \frac{\theta\left[\eta+\beta(1-\alpha)\right] n_{t-1}}{\beta(1-\alpha)\phi\theta^2 + (1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]n_{t-1}} \tag{19}$$

If $\eta+\beta(1-\alpha)(1-\phi\theta) > 0$, then n_t monotonically converges to

$$n^N = \frac{\theta[\eta+\beta(1-\alpha)(1-\phi\theta)]}{(1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]} \tag{20}$$

The gift rate and the saving rate converge monotonically to

$$g^N = \frac{\beta\phi\theta - \alpha(\eta+\beta\phi\theta)}{(1-\alpha)(\beta+\eta)} \tag{21}$$

$$s^N = \frac{\alpha[\eta+\beta(1-\alpha)(1-\phi\theta)]}{(1-\alpha)(1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]} \tag{22}$$

The interior solution requires

$$\frac{\alpha\eta}{\beta(1-\alpha)\phi} < \theta < \frac{\eta+\beta(1-\alpha)}{\beta(1-\alpha)\phi}$$

Proof. See Appendix. ■

The process of capital accumulation is complicated by the fertility dynamics. From equation (10), we have

$$k_{t+1} = \frac{\alpha}{\theta}\frac{\Phi(n_t)}{\Phi(n_{t-1})}f(k_t) \tag{23}$$

where

$$\Phi(n_t) = 1 + \beta + \eta + \frac{\beta\theta}{n_t} \tag{24}$$

Assume that the fertility rate in period $t-1$ is higher than the steady state, $n_{t-1} > n^N$. First, we know $n_{t-1} > n_t > n^N$ from equation (19). Second, we know $\Phi(n_t) > \Phi(n_{t-1})$ from equation (24). Therefore the coefficient of $f(k_t)$ in the right hand side of equation (23) is larger than α/θ in the process of transition. Finally, using equation (19), we have

$$\frac{\Phi(n_t)}{\Phi(n_{t-1})} = \frac{\beta^2(1-\alpha)\phi\theta^2 + (1+\beta+\eta)[\beta+\eta+\beta(1-\alpha)\phi\theta]n_{t-1}}{[\eta+\beta(1-\alpha)][\beta\theta+(1+\beta+\eta)n_{t-1}]}$$

which is increasing in n_{t-1}. Therefore, the coefficient decreases monotonically and converges to α/θ in the process of fertility decline. The law of motion of k_t depends not only on the downward shift of the investment curve but also on the initial condition.

[Figure 2 and 3 are here]

Figure 2 illustrates a case in which capital accumulation is non-monotonic. Starting at a low initial condition, the capital-labor ratio increases steadily and may go far beyond the steady state at any given time. Then, it decreases and converges to the steady state. Figure 3 illustrates the relationship between fertility and capital accumulation in the non-cooperative siblings model.

3.3 Comparison

In this section, we compare the steady-state equilibrium when siblings are cooperative and when they are not. The following propositions summarize the results.

Proposition 3 *For a small child-rearing cost ϕ, there exists a pair of $(\underline{\theta}, \overline{\theta})$ such that $n^C > 1$ and $n^N > 1$ for any $\theta \in (\underline{\theta}, \overline{\theta})$.*

Proof. See Appendix. ■

Proposition 4 *For any $\theta \in (\underline{\theta}, \overline{\theta})$, we have*
(i) $g^N < g^C$
(ii) $s^N > s^C$
(iii) $k^N > k^C$

Proof. See Appendix. ■

The reason is simple. First, other things be equal, non-cooperative siblings evaluate their own gifts less than cooperative ones if the number of siblings is strictly larger than one. Therefore, the steady-state gift rate in the non-cooperative siblings model is lower than that in the cooperative siblings model. Second, the lower the gift rate is, the higher the saving rate. A lower gift rate implies that disposable income in the working period is larger, and that disposable income in the retirement period is smaller. Therefore, individuals have incentives to save in order to smooth their consumption. Third, the higher is the saving rate, the higher the steady state capital-labor ratio. Comparing the steady states, the wage rate is higher and the interest rate is lower in the non-cooperative siblings model.

One might imagine that the steady state fertility rate in the non-cooperative siblings model is always lower than the fertility rate in the cooperative siblings model because individuals have to increase private savings. The answer is no. From equations (A11) and (A12) in Appendix, we have

$$\frac{n^C - 1}{n^N - 1} = \frac{1 + \beta + \eta}{1 + \beta + \eta + \beta\theta}\left[1 + \frac{\alpha}{\phi\theta(1-\alpha)}\right] \tag{25}$$

Because equation (25) is decreasing in θ, it can be true that $n^N > n^C > 1$ for a large θ. Specifically, we have the following proposition.

Proposition 5 *Assume that $\theta \in (\underline{\theta}, \overline{\theta})$. Then, $n^N > n^C > 1$ for any $\theta > \tilde{\theta}$ and $n^C > n^N > 1$ for any $\theta < \tilde{\theta}$, where*

$$\tilde{\theta} = \sqrt{\frac{\alpha(1 + \beta + \eta)}{(1-\alpha)\beta\phi}} \tag{26}$$

4 Dynamic efficiency

One of the most controversial issues in the gift economy is that the steady-state equilibrium tends to be dynamically inefficient (Abel (1987), O'Connell and Zeldes (1993)). Because our model includes a choice of fertility, one might imagine that the assumption of endogenous fertility is critical to the dynamic efficiency condition. We show, however, that it is not fertility, but rather strategic behavior among siblings, that affects the dynamic efficiency condition.

From equations (9) and (10), we have, at a steady state,

$$\frac{1 + r}{n} = \frac{\alpha}{1 - \alpha}\frac{1}{s} \tag{27}$$

Equation (27) provides a simple rule for the dynamic efficiency condition: the economy is dynamically efficient if and only if the equilibrium saving rate, s, is smaller than the ratio of the share of capital income to the share of labor income, $\alpha/(1-\alpha)$. Specifically, we have the following proposition.

Proposition 6 *In the cooperative siblings model, the steady-state equilibrium is dynamically efficient if and only if $\theta > 1$.*

In the non-cooperative siblings model, the steady-state equilibrium is dynamically efficient if and only if

$$\theta > \frac{\beta + \eta - \alpha(1 + 2\beta + \eta)}{\phi(1-\alpha)(1 + 2\beta + \eta)} \tag{28}$$

Proof. Substituting s^C in equation (16) into equation (27), we have $(1+r)/n = \theta$. Therefore $\theta > 1$ is a necessary and sufficient condition for dynamic efficiency.

Substituting s^N in equation (22) into equation (27), we have

$$\frac{1+r}{n} = \frac{(1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]}{\eta+\beta(1-\alpha)(1-\phi\theta)}$$

which is larger than one if and only if equation (28) is satisfied. ■

The condition $\theta > 1$ is well-known and often criticized because altruism to parents is too strong (Michel et al. (2006)). The condition in equation (28) is one of the contributions of this paper. Suppose that the preference for the number of children is weak enough to make the right hand side of equation (28) negative, that is,

$$\eta < \frac{\alpha}{1-\alpha}(1+\beta) - \beta \tag{29}$$

Then, the steady-state equilibrium in the non-cooperative siblings model is dynamically efficient for all $\theta > 0$, given that gifts are operative[3]. The reason is simple. The smaller η is, the smaller the equilibrium fertility rate. Because the free-rider problem is alleviated, children increase giving gifts to their parent, which in turn decreases the parent's private saving because they expect to receive gifts from their children in the future. Therefore, the interest rate tends to increase beyond the depressed fertility rate in the capital market.

5 Conclusions

While intergenerational transfers from parents to children have been extensively analyzed in public economics and population economics, intergenerational transfers from children to parents have not been well developed, especially in the dynamic context. One of the reasons could be a theoretical limitation insofar as gift economies tend to be dynamically inefficient. Alternatively, unrealistic altruism towards the parent must be assumed to make gifts operative. By incorporating non-cooperative behavior among siblings into a simple overlapping generations model, we tried to overcome this theoretical limitation. In a non-cooperative siblings model, we show that the fertility rate declines over time, capital accumulation can be non-monotonic, and the steady-state equilibrium tends to be dynamically efficient. Further investigation into strategic behaviors within families could be worth-while to make the gift economy model applicable to issues in public economics and population economics.

[3] The condition (29) is not so restrictive. Assume that the capital share is $\alpha = 1/3$ and that the private discount factor is $\beta = 2/3$ (which implies an annual discount rate is 1.3 per cent when one period is 30 years). Then, equation (29) implies $\eta < 1/6$.

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Appendix
[Proof of Proposition 2]

From equation (18),

$$g_t = \frac{\beta\theta - \frac{\alpha}{1-\alpha}(1+\beta+\eta)n_{t-1}}{\beta\theta + (1+\beta+\eta)n_{t-1}} \tag{A1}$$

From equations (6) and (11),

$$s_t = \frac{\beta(1-g_t)}{(1+\beta+\eta)\left(1+\frac{1-\alpha}{\alpha}g_{t+1}\right)} \tag{A2}$$

$$\phi n_t = \frac{1-g_t}{1+\beta+\eta}\left(\beta+\eta-\frac{\beta}{1+\frac{1-\alpha}{\alpha}g_{t+1}}\right) \tag{A3}$$

First, substituting equation (A1) into equation (A3), we have

$$n_t = \frac{\theta[\eta+\beta(1-\alpha)]n_{t-1}}{\beta(1-\alpha)\phi\theta^2 + (1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]n_{t-1}} \tag{A4}$$

which is equation (19) in the main body.

Let us define a function of n by

$$f(n) = \frac{An}{B+Cn}$$

where A, B, and C are all positive constants. The function is increasing and concave in $n \geq 0$ and has a upper bound of A/C.

Equation $n = f(n)$ has a unique positive solution if and only if

$$f'(0) > 1 \Leftrightarrow A > B$$

From equation (A4), this condition is equivalent to

$$\eta + \beta(1-\alpha)(1-\phi\theta) > 0 \tag{A5}$$

If equation (A5) is satisfied, then n_t monotonically converges to

$$n^N = \frac{\theta\left[\eta+\beta(1-\alpha)(1-\phi\theta)\right]}{(1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]} \tag{A6}$$

which is equation (20) in the main body.

Second, equation (A1) implies the gift rate monotonically converges to

$$\begin{aligned} g^N &= \frac{\beta\theta - \frac{\alpha}{1-\alpha}(1+\beta+\eta)n^N}{\beta\theta + (1+\beta+\eta)n^N} \\ &= \frac{\beta\phi\theta - \alpha(\eta+\beta\phi\theta)}{(1-\alpha)(\beta+\eta)} \end{aligned} \tag{A7}$$

which is equation (21) in the main body. The interior condition requires

$$\theta > \frac{\alpha\eta}{\beta\phi(1-\alpha)}$$

Finally, we examine the law of motion of the saving rate. Substituting equation (A1) into equation (A2), we have

$$s_t = \frac{\alpha n_{t-1}}{(1-\alpha)\theta} \cdot \frac{\beta\theta + (1+\beta+\eta)n_t}{\beta\theta + (1+\beta+\eta)n_{t-1}} \tag{A8}$$

Therefore the saving rate converges to

$$\begin{aligned} s^N &= \frac{\alpha n^N}{(1-\alpha)\theta} \\ &= \frac{\alpha[\eta + \beta(1-\alpha)(1-\phi\theta)]}{(1-\alpha)(1+\beta+\eta)[\alpha + (1-\alpha)\phi\theta]} \end{aligned} \tag{A9}$$

which is equation (22) in the main body.

To examine the dynamics, substituting equation (A4) into equation (A8), we have

$$s_t = \frac{\alpha n_{t-1}\left\{\beta^2(1-\alpha)\phi\theta^2 + (1+\beta+\eta)[\beta+\eta+\beta(1-\alpha)\phi\theta]n_{t-1}\right\}}{(1-\alpha)[\beta\theta + (1+\beta+\eta)n_{t-1}]\left\{\beta(1-\alpha)\phi\theta^2 + (1+\beta+\eta)[\alpha + (1-\alpha)\phi\theta]n_{t-1}\right\}} \tag{A10}$$

Log-differentiating equation (A10) with respect to n_{t-1}, we have

$$\begin{aligned} \frac{\partial \ln s_t}{\partial n_{t-1}} &= \frac{1}{n_{t-1}} - \frac{1+\beta+\eta}{\beta\theta + (1+\beta+\eta)n_{t-1}} \\ &\quad + \frac{(1+\beta+\eta)[\beta+\eta+\beta(1-\alpha)\phi\theta]}{\beta^2(1-\alpha)\phi\theta^2 + (1+\beta+\eta)[\beta+\eta+\beta(1-\alpha)\phi\theta]n_{t-1}} \\ &\quad - \frac{(1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]}{\beta(1-\alpha)\phi\theta^2 + (1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]n_{t-1}} \end{aligned}$$

It can be shown that the sum of the first and second terms is positive, and that the sum of the third and fourth term is also positive. Thus, we know $\partial s_t/\partial n_{t-1} > 0$ for any n_{t-1}. Because the path of n_t is monotonic, the path of s_t is also monotonic.

[Proof of Proposition 3]

From equations (14) and (20), we have

$$n^C - 1 = \frac{(\beta+\eta)\theta - (1+\beta+\eta+\beta\theta)[\alpha+(1-\alpha)\phi\theta]}{\phi\theta(1-\alpha)(1+\beta+\eta+\beta\theta)} \tag{A11}$$

$$n^N - 1 = \frac{(\beta+\eta)\theta - (1+\beta+\eta+\beta\theta)[\alpha+(1-\alpha)\phi\theta]}{(1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]} \tag{A12}$$

Therefore the steady-state fertility rates are larger than one if and only if

$$(\beta+\eta)\theta - (1+\beta+\eta+\beta\theta)[\alpha+(1-\alpha)\phi\theta] > 0 \tag{A13}$$

Rearranging terms, equation (A13) is a quadratic inequality of θ such as

$$\beta(1-\alpha)\phi\theta^2 - [\eta + \beta(1-\alpha) - (1+\beta+\eta)(1-\alpha)\phi]\theta + \alpha(1+\beta+\eta) < 0 \tag{A14}$$

Denote the discriminant by D,

$$D = [\eta + \beta(1-\alpha) - (1+\beta+\eta)(1-\alpha)\phi]^2 - 4\beta\alpha(1-\alpha)(1+\beta+\eta)\phi$$

which is positive for a small ϕ.

Then, condition (A14) is satisfied for any $\theta \in (\underline{\theta}, \overline{\theta})$, where

$$\underline{\theta} = \frac{\eta + \beta(1-\alpha) - (1+\beta+\eta)(1-\alpha)\phi - \sqrt{D}}{2\beta(1-\alpha)\phi}$$
$$\overline{\theta} = \frac{\eta + \beta(1-\alpha) - (1+\beta+\eta)(1-\alpha)\phi + \sqrt{D}}{2\beta(1-\alpha)\phi}$$

[Proof of Proposition 4]

From equations (15) and (21), we have

$$g^C - g^N = \frac{\beta\left\{(\beta+\eta)\theta - (1+\beta+\eta+\beta\theta)[\alpha+(1-\alpha)\phi\theta]\right\}}{(1-\alpha)(\beta+\eta)(1+\beta+\eta+\beta\theta)}$$

which is positive if equation (A13) is satisfied. Therefore, $g^C > g^N$ for any $\theta \in (\underline{\theta}, \overline{\theta})$.

From equations (16) and (22), we have

$$s^C - s^N = -\frac{\alpha\left\{(\beta+\eta)\theta - (1+\beta+\eta+\beta\theta)[\alpha+(1-\alpha)\phi\theta]\right\}}{(1-\alpha)\theta(1+\beta+\eta)[\alpha+(1-\alpha)\phi\theta]}$$

which is negative if equation (A13) is satisfied. Therefore, $s^N > s^C$ for any $\theta \in (\underline{\theta}, \overline{\theta})$.

Comparing the coefficients of $f(k_t)$ in equations (17) and (23), we have

$$\begin{aligned} & \frac{\alpha}{\theta} - \frac{\phi\alpha(1-\alpha)(1+\beta+\eta+\beta\theta)}{(\beta+\eta)\theta - \alpha(1+\beta+\eta+\beta\theta)} \\ = \quad & \frac{\alpha\left\{(\beta+\eta)\theta - (1+\beta+\eta+\beta\theta)[\alpha+(1-\alpha)\phi\theta]\right\}}{\theta[(\beta+\eta)\theta - \alpha(1+\beta+\eta+\beta\theta)]} \end{aligned}$$

which is positive if equation (A13) is satisfied. Therefore, $k^N > k^C$ for any $\theta \in (\underline{\theta}, \overline{\theta})$.

Figure 1. Fertility dynamics and capital accumulation in the cooperate siblings model

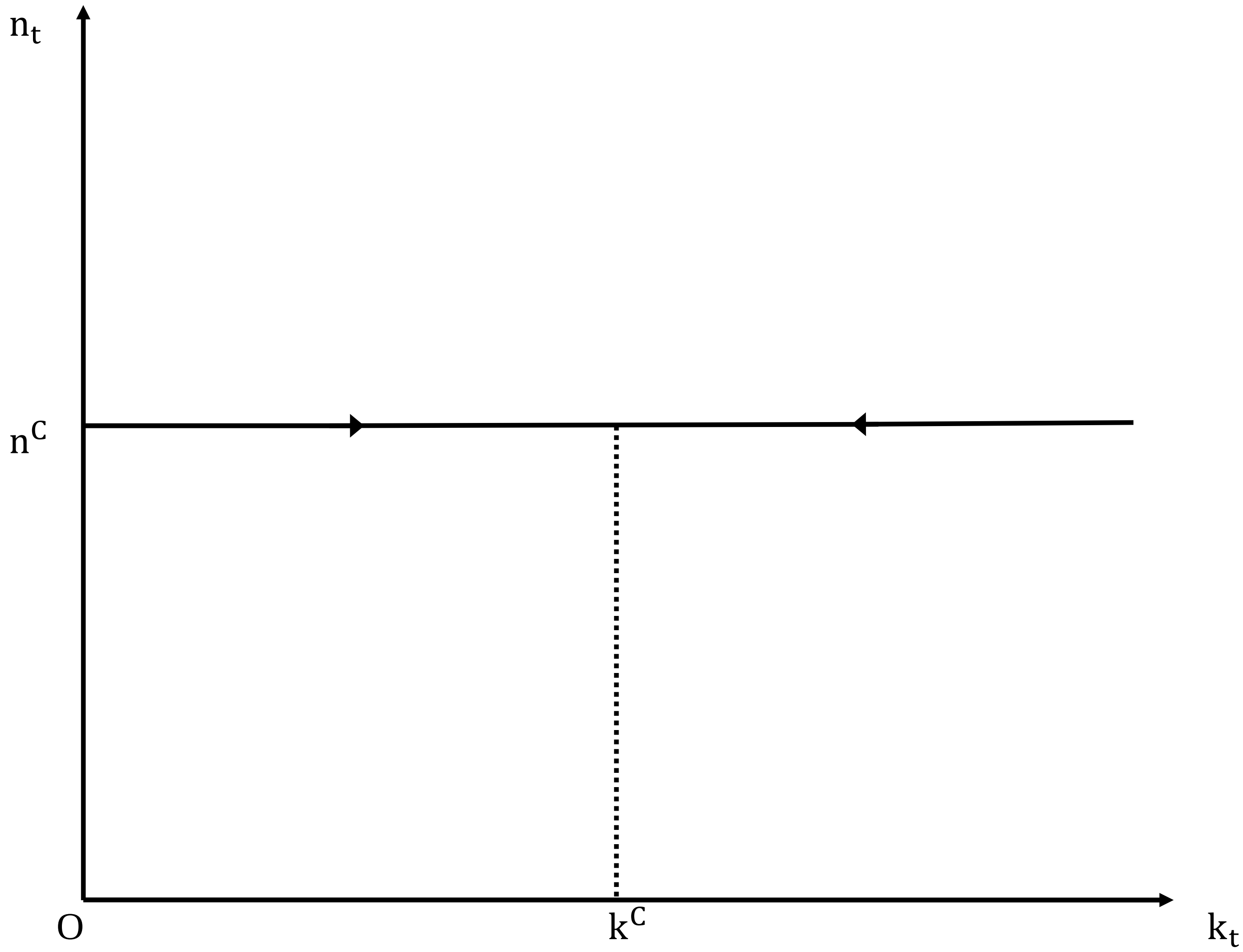

Figure 2. Capital accumulation in the non-cooperative siblings model

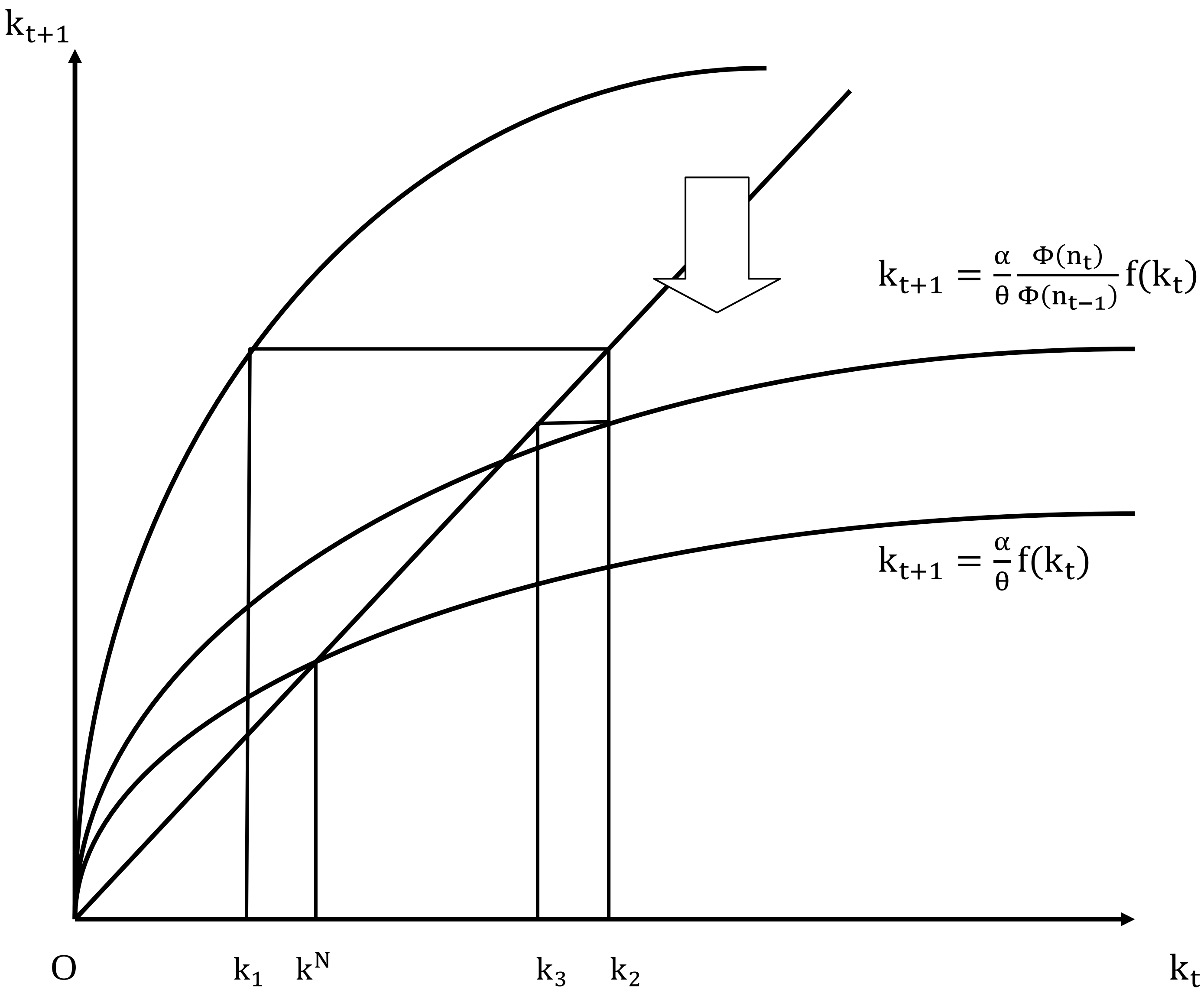

Figure 3. Fertility dynamics and capital accumulation in the non-cooperate siblings model