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Abstract

Population aging may be an opportunity in a sense that it creates a demand for less durable goods (services) and that it creates a job for skilled older workers in the service sector. In this changing demographic and economic environment, we show that public pensions effectively achieve a social optimum by reallocating unskilled labor from a manufacturing to the service sector.

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1 Introduction

The purpose of the paper is to give an answer to the question of whether aging society is opportunity or challenge (Cutler et al. (1990)). Some people may have a negative opinion in that an increase in the dependency ratio imposes financial burdens on young workers, which discourages work incentives. Others may believe that an increase in the political power of older people makes public expenditures in favor of older rather than younger people. Such discussions depend on whether older people are economically inactive or not. In fact many OECD countries have experienced a rise in the proportion of older people aged 50-64 who are working since the mid-1990s (OECD (2006)).[1] If older people continue to work in the labor market, then they contribute to an expansion of production possibility set, which not only improves their own welfare but also may improve the welfare of younger people.

We interpret population aging as a continuous decline in an adult mortality rate in a two-period overlapping generations model. It is assumed that one period is 40 years, and that each individual in each generation enters the model economy when he becomes 20 years old. Thus, supposed life expectancy at birth varies from 60 years to 100 years.[2]

In our model population aging affects economic outcomes through three channels. First, it encourages capital accumulation because young people save more than before for their prolonged second period of life. Since capital deepening attracts labor from a labor intensive sector to a capital intensive sector, the capital deepening effect changes the allocation of labor. Second, population aging changes demand components. Older people would like to consume less durable goods (say, services) because their longevity is shorter than younger people. The increased demand for services attracts the demand for labor in service production sectors. Assuming that the service sectors are more labor intensive than good production sectors, the demand effect reallocates labor in the opposite direction of the capital deepening effect.

Finally, and most importantly, population aging increases an opportunity for older people to work in the emerging service sectors. One of the reasons is that older people have an advantage over younger counterparts in management jobs because they have experienced longer time in the workplace and accumulated skills for management. Second, some kind of service sectors may well be more flexible and less physically demanding than manufacturing sectors, which improves employability for older workers. We assume that old-age labor and young-age labor are not substitutes but complements.[3] In this setting, an increase in the employment of old-age labor attracts young-age labor from good

[1] There are large differences between OECD countries in both the level of and the increase in the labor force participation rates among older people. In 2004, the level varies from over 80 per cent (Iceland) to 40 per cent (Turkey). In a decade from 1994 to 2004, Netherlands, New Zealand, and Belgium have experienced over 10 percentage point increase, but the rate is almost constant in Japan, Korea, and Austria (ibid, p.29).

[2] There are also large differences between OECD countries in life expectancy at birth. In 2000, it varies from 81.2 (Japan) to 68.1 (Turkey). The avearge among OECD countries is 77.0. In 2050, it is expected that the average is 83.4 and that the difference shrinks (from 87.7 (France) to 78.5 (Turkey)) (ibid. p.18).

[3] OECD (2006) suggests a countercharge against the lump of labor fallacy, which implies there are a fixed number of jobs to go around and workers are perfectly substitutable for each other. Across OECD countries, there is a positive rather than negative correlation between changes in employment rates for younger and older people (Figure 7.1, p.140).

production sectors to service sectors. Thus, the labor supply effect enhances the demand effect mentioned above.

We introduce a pay-as-you-go public pension into the model economy, and analyze whether the public pension is a useful device to achieve a social optimum. We show (1) when the average longevity exceeds a threshold (about 63 years old), the public pension effectively achieves a socially optimal allocation by reallocating young-age (unskilled) labor from manufacturing to service sectors, and that (2) the optimal contribution rate increases almost linearly with average longevity. For example, it is about 8 per cent when the longevity is 77.0, and about 20 per cent when the longevity is 83.4.

Our model is relevant to some economic and policy debates. First, our model explains the Petty-Clark's law in a demographic point of view. In terms of employment as well as value-added, the share of service sector increases in a process of population aging. Acemoglu and Guerrieri (2008) presents an alternative model to explain the scenario. Assuming a complementarity between a capital intensive sector and a labor intensive sector and different rates of technical progress between sectors, it shows both capital and labor move from the former to the latter sector in a process of capital deepening. The reason is simple. Suppose that the rate of technical progress in the capital intensive sector is higher than the labor intensive sector. Since two goods are complements, an aggregate steady growth requires more resources in the labor intensive sector. We admit our model also depends on the complementarity, but the mechanism is fairly different. We assume a complementarity between old-age skilled labor and young-age unskilled labor. Further our result fits more for the reallocation of labor.[4]

Second, our model gives some insights into the field of research on service economy. In a growth model in which a good is produced by capital and labor, a service is produced by labor only, and the preference to service increases with age, van Groezen et al. (2005) shows that population aging enhances economic growth in a closed economy if the elasticity of substitution between capital and labor in the production sector is less than one. The fundamental mechanism is similar to Acemoglu and Guerrieri (2008). The paper assumes the service sector employs young-age labor only, so that the labor supply effect in our model is not examined. The paper incorporates a pay-as-you-go public pension scheme, but the optimality is not discussed. In a similar model, van Groezen et al. (2007) shows that the golden rule capital stock in a two-sector model is lower than in a one-sector model. The paper gives a numerical example that, when the elasticity of substitution between goods and services is relatively small, the market economy is dynamically inefficient for a large set of parameter values (Fig.1, p.157). It suggests the rate of contribution to public pensions that achieves the golden rule is positive, and that an increase in the dependency ratio increases the contribution rate. However, the rates indicated in the paper are fairly small (Table 2, p.162). Further the golden rule criterion is necessary but not sufficient for optimality considerations. To discuss the optimality, we need examine whether the allocation of labor and consumption are also optimal. Our model shows that there exists a unique optimal contribution rate in some circumstances.

[4]Acemoglu and Guerrieri (2008) indicates the ratio of employment in the labor intensive sector to that in the capital intensive sector is 1.03 in 1948 and 0.80 in 2005. The decline is about 33 per cent. The basic model in the paper explains only 5 per cent (p.491).

Third, our model presents income components for older people on an optimality basis.[5] It predicts that the share of labor income is 20 per cent regardless of longevity, and that pension benefits crowd out capital income as the average longevity increases. For example, the share of pension benefits is 20 per cent when the longevity is 77.0, and about 37 per cent when the longevity is 83.4.

Finally, some argue that the government should reform the existing public pension scheme because it discourages work incentives for old-age workers and induces early retirement. In an extended model which incorporates a retirement decision into the basic model, we show the labor participation rate of old-age workers indeed decreases with longevity and the contribution rate. Our model suggests that the rate of suspension related to work at an older age should be lowered gradually in a process of population aging.

The structure of the paper is as follows. In section 2 we introduce a basic model. In section 3 we first derive the equilibrium and then examine whether and to what extent pay-as-you-go public pensions are effective in achieving a socially optimal allocation. In section 4 we extends the basic model to deal with early retirement and employability for older workers. The final section concludes the paper.

2 The basic model

2.1 Setup

We use a two-period overlapping generations model with an adult mortality risk. In each period newly born individuals enter the economy. The population size is normalized to unity. They live for two period at most. They face a mortality risk when they enter the second period of life. The survival probability is denoted by $\theta \in (0,1]$. In the first period of life, each individual supplies one unit of unskilled labor either in the good production sector or in the service production sector to earn wage income. He consumes a (durable) good, purchases actuarially fair private annuities, and contributes to a pay-as-you-go public pension. If he survives into the second period, he works as a manager in the service sector to earn management income. He also receives the return of private annuities and the public pension benefit to consume the service provided. Our model contrasts with usual two-period overlapping generations models in that individuals that survive into the second period are economically active. They have not only a chance to enjoy consuming services but also a chance to get money as managers in their later stage of life.

The utility of an individual who is born at period t is specified by

$$u_t = \ln c_t + \gamma\theta \ln d_{t+1} \tag{1}$$

where c_t and d_{t+1} stand for the good consumption in the first period and the service consumption in the second period, respectively. $0 < \gamma \leq 1$ is a private discount factor.

[5] OECD (2009) shows the share of public transfers, labor income, and capital income in some OECD countries. The share of public transfers is higher in France (85%), Belgium (81%), and Austria (80%). The share of labor income is higher in Korea (59%), Japan (44%), and United States (34%).

The budget constraints are

$$(1-\tau)w_t = c_t + s_t \tag{2}$$
$$R_{t+1}s_t + \pi_{t+1} + P_{t+1} = q_{t+1}d_{t+1} \tag{3}$$

Equation (2) stands for the budget constraint in the first period. w_t is a wage rate and $\tau \in [0,1)$ is a rate of contribution to the public pension. s_t stands for the purchase of private annuities. Equation (3) stands for the budget constraint in the second period. R_{t+1} is a rate of return of private annuities and P_{t+1} is a public pension benefit. π_{t+1} stands for the management income, and q_{t+1} stands for a price of the service.[6]

Each individual chooses c_t, d_{t+1}, and s_t to maximize equation (1) subject to equations (2) and (3). Solving the problem, we have

$$c_t = \frac{1}{1+\gamma\theta} I_t \tag{4}$$
$$d_{t+1} = \frac{\gamma\theta}{1+\gamma\theta}\frac{R_{t+1}I_t}{q_{t+1}} \tag{5}$$
$$s_t = \frac{\gamma\theta}{1+\gamma\theta}(1-\tau)w_t - \frac{1}{1+\gamma\theta}\frac{\pi_{t+1}+P_{t+1}}{R_{t+1}} \tag{6}$$

where I_t stands for lifetime full income,

$$I_t = (1-\tau)w_t + \frac{\pi_{t+1}+P_{t+1}}{R_{t+1}}$$

From equation (6), the purchase of private annuities is positive when

$$\gamma\theta(1-\tau)w_t > \frac{\pi_{t+1}+P_{t+1}}{R_{t+1}} \tag{7}$$

Hereafter, we assume equation (7) is satisfied[7].

In the good production sector, competitive firms employ capital and unskilled labor to produce a homogeneous good. The technology is specified by a Cobb-Douglas production function,

$$y_{1t} = f(k_t, l_{1t}) = Ak_t^{\alpha} l_{1t}^{1-\alpha} \tag{8}$$

where y_{1t}, k_t, and l_{1t} stand for the output, capital, and unskilled labor employed in the good sector, respectively. $0 < \alpha < 1$ is a share of capital income, and $A > 0$ is a productivity parameter.

Competition in the factor market requires

$$w_t = (1-\alpha)Ak_t^{\alpha} l_{1t}^{-\alpha} \tag{9}$$
$$1 + r_t = \alpha A k_t^{\alpha-1} l_{1t}^{1-\alpha} \tag{10}$$

We assume capital is fully depreciated in one period.

[6] We assume all individuals that survive into the second period of life continue to work as managers. The problem of early retirement is discussed in Section 4.1.

[7] We verify it is true in equilibrium below.

In the service production sector, competitive firms employ managers and unskilled labor to produce a homogeneous service[8]. The technology is specified by a Cobb-Douglas production function,

$$y_{2t} = g(h_t, l_{2t}) = Bh_t^{\beta} l_{2t}^{1-\beta} \tag{11}$$

where y_{2t}, h_t, and l_{2t} stand for the output, the number of managers, and unskilled labor employed in the service sector, respectively. $0 < \beta < 1$ is a share of management income, and $B > 0$ is a productivity parameter.

Competition in the factor market requires

$$w_t = q_t(1-\beta)Bh_t^{\beta} l_{2t}^{-\beta} \tag{12}$$

$$\pi_t = q_t\beta Bh_t^{\beta-1} l_{2t}^{1-\beta} \tag{13}$$

where q_t is a price of the service, and π_t is the management income.

In our model, there are 6 markets, that is, markets of unskilled labor, managers, capital, private annuities, goods and services. The following equations stand for the corresponding market clearing conditions.

$$l_{1t} + l_{2t} = 1 \tag{14}$$

$$h_t = \theta \tag{15}$$

$$k_{t+1} = s_t \tag{16}$$

$$R_t = \frac{1+r_t}{\theta} \tag{17}$$

$$y_{1t} = c_t + k_{t+1} \tag{18}$$

$$y_{2t} = \theta d_t \tag{19}$$

Finally, the balanced budget condition for the public pension is given by

$$\tau w_t = \theta P_t \tag{20}$$

The Walras law tells us one of the conditions is redundant. It can be shown that both the good market and the service market clear if and only if[9]

$$(1+r_t)k_t = w_t l_{2t} - \tau w_t \tag{21}$$

2.2 Equilibrium

In this section we derive the equilibrium and examine the characteristics of key variables.

Since the income share is constant, the capital income is proportional to the labor income: $(1+r_t)k_t = \frac{\alpha}{1-\alpha} w_t l_{1t}$. Substituting this into equation (21), we have

$$l_{2t} = \frac{\alpha}{1-\alpha} l_{1t} + \tau$$

[8] It may be possible that managers are employed in the good production sector. We discuss it in Section 4.2.

[9] Substituting equations (2) and (16) into equation (18), we have $y_{1t} = (1-\tau)w_t$. From equation (14), the capital income is given by $(1+r_t)k_t = (1-\tau)w_t - w_t l_{1t} = w_t l_{2t} - \tau w_t$.
Substituting equation (3) into equation (19), and using equations (12), (13), (17), and (20), we have the same equation.

which implies the public pension moves unskilled labor from the good sector to the service sector. From the labor market condition (14), the allocation of unskilled labor is given by

$$l_1 = (1-\alpha)(1-\tau) \tag{22}$$

$$l_2 = \alpha(1-\tau)+\tau \tag{23}$$

From equations (9), (10), and (22), we know

$$\frac{w_{t+1}}{1+r_{t+1}} = \frac{k_{t+1}}{\alpha(1-\tau)} \tag{24}$$

From equations (17), (20), and (24), the present value of the pension benefit is given by

$$\frac{P_{t+1}}{R_{t+1}} = \frac{\tau}{\alpha(1-\tau)}k_{t+1} \tag{25}$$

From equations (12), (13), (17), and (24), the present value of the management income is given by

$$\frac{\pi_{t+1}}{R_{t+1}} = \frac{\beta}{1-\beta}\left[1+\frac{\tau}{\alpha(1-\tau)}\right]k_{t+1} \tag{26}$$

Substituting equations (25) and (26) into equation (6), and using equation (16), we have

$$k_{t+1} = \frac{\alpha\gamma\theta(1-\tau)w_t}{\alpha\gamma\theta+\frac{1}{1-\beta}\left(\alpha+\frac{\tau}{1-\tau}\right)} \tag{27}$$

which regulates the law of motion of k_t (with equation (9)).

Since $y_{1t}=(1-\tau)w_t$, equation (27) tells us the investment ratio is constant over time,

$$\frac{k_{t+1}}{y_{1t}} = \frac{\alpha\gamma\theta}{\alpha\gamma\theta+\frac{1}{1-\beta}\left(\alpha+\frac{\tau}{1-\tau}\right)} \tag{28}$$

The dynamics is simple. Given that τ and θ are constant over time, the allocation of unskilled labor is constant (equations (22) and (23)). The investment ratio is also constant (equation (28)). Given an initial capital stock, the capital stock converges monotonically to a unique and stable steady state (equations (9) and (27)). In a process of capital accumulation, the wage rate increases, which induces both the price of the service, q_t, and the management income, π_t, to increase (equations (12) and (13)).

Proposition 1 *The share of capital income, pension benefits, and labor income of old-age workers are respectively given by*

$$\frac{R_t s_{t-1}}{q_t d_t} = \frac{(1-\beta)\alpha(1-\tau)}{\alpha(1-\tau)+\tau}$$

$$\frac{P_t}{q_t d_t} = \frac{(1-\beta)\tau}{\alpha(1-\tau)+\tau}$$

$$\frac{\pi_t}{q_t d_t} = \beta$$

The income shares are independent of the survival rate, θ. The capital share is decreasing in τ, and the labor share is independent of τ.

Proof. Straightforward from equations (3), (25), and (26). ■

At the end of this section, we briefly prove the interior condition (7) is satisfied in equilibrium. From equations (25), (26), and (27), we know

$$\frac{\pi_{t+1}+P_{t+1}}{R_{t+1}} = \left[1 - \frac{\alpha(1+\gamma\theta)}{\alpha\gamma\theta + \frac{1}{1-\beta}\left(\alpha + \frac{\tau}{1-\tau}\right)}\right]\gamma\theta(1-\tau)w_t$$

which is smaller than $\gamma\theta(1-\tau)w_t$ for $\forall\tau \in [0,1)$ and $\forall\theta \in (0,1]$.

3 An optimal pension scheme

In the previous section we treat the rate of contribution to the public pension, τ, as if it is independent of demographic factors. It seems more insightful to relate the pension scheme to a demographic factor in a meaning way.

Appendix A shows the optimal employment in the good sector is given by

$$l_1^* = \frac{\rho(1-\alpha)}{\rho(1-\alpha)+(1-\rho\alpha)(1-\beta)\gamma\theta} \tag{29}$$

and the optimal capital-output ratio is given by

$$\left(\frac{k_{t+1}}{y_{1t}}\right)^* = \rho\alpha \tag{30}$$

where $0 < \rho < 1$ is a social discount factor. An asterisk is used for the optimal variables.

Comparing equations (22) and (29), we have a contribution rate that achieves the optimal allocation of unskilled labor. Comparing equations (28) and (30), we have a contribution rate that achieves the optimal resource allocation. If the both rates coincide with each other, the public pension could achieve the optimal allocation. The following proposition tells us it is the case.

Proposition 2 *The decentralized economy is optimal if and only if the rate of contribution to the public pension is*

$$\tau^* = \frac{(1-\rho\alpha)(1-\beta)\gamma\theta - \rho\alpha}{\rho(1-\alpha)+(1-\rho\alpha)(1-\beta)\gamma\theta} \tag{31}$$

Proof. Straight calculation shows, under the contribution rate τ^* in equation (31), equation (22) is equal to equation (29), and equation (28) is equal to equation (30). ■

The reason is simple. Let us define the industry ratio by

$$IR_t = \frac{q_t y_{2t}}{y_{1t}}$$

In our model, the industry ratio in the competitive equilibrium is constant over time,

$$IR = \frac{1}{1-\beta}\left(\alpha + \frac{\tau}{1-\tau}\right)$$

Appendix A shows the optimal industry ratio is given by

$$IR^* = \frac{(1-\rho\alpha)\gamma\theta}{\rho}$$

It can be easily shown that $IR = IR^*$ is equivalent to $l_1 = l_1^*$ and that the Euler equation,

$$\frac{c_{t+1}}{c_t} = \rho(1 + r_{t+1})$$

is satisfied if $IR = IR^*$. Therefore, the adjustment of the industry ratio makes both intratemporal and intertemporal allocation optimal in our model.

From equation (31), we know $\tau^* \geq 0 \Leftrightarrow \theta \geq \hat{\theta}$ where

$$\hat{\theta} = \frac{\rho\alpha}{\gamma(1-\rho\alpha)(1-\beta)} \tag{32}$$

A pay-as-you-go public pension effectively achieves the optimal allocation if the survival probability exceeds the threshold in equation (32). The reason is simple. Equation (29) tells us, for a larger θ, the optimal employment in the good sector is smaller because a larger θ makes the optimal size of the service sector larger. Public pensions serve the purpose by reallocating unskilled labor from the good sector to the service sector. Roughly speaking, public pensions assist industrial restructuring induced by population aging.

From equation (32), we know $\hat{\theta}$ is increasing in ρ, α, and β, and decreasing in γ. In our model public pensions are more likely to be effective when $\hat{\theta}$ is smaller. It is the case when society attaches more importance to current generations rather than future generations, the income share of unskilled labor is larger, or individuals are more patient. The following proposition summarizes the results.

Proposition 3 *In an aging economy ($\theta \geq \hat{\theta}$), pay-as-you-go public pensions effectively achieve an optimal allocation. The threshold survival rate $\hat{\theta}$ is lower when the social discount factor is smaller, when the income share of unskilled labor is larger, or when the private discount factor is larger.*

[Figure 1 is here]

Let us provide a numerical example. Suppose that individuals start their economic activity at 20 years old, and that one period is 40 years. Assuming that an annual rate of time preference is one per cent, we have $\gamma = 1.01^{-40} = 0.67$. The income shares are $\alpha = 0.3$ and $\beta = 0.2$, which implies the service sector uses unskilled labor more intensively than the good production sector. The social discount factor is $\rho = 0.5$. Equation (32) gives $\hat{\theta} = 0.33$, which implies the average longevity is $20 + 40 * 1.33 = 73.2$. Figure 1 illustrates the optimal contribution rate which is given by equation (31). The horizontal axis is measured by the average longevity instead of θ. We know the optimal rate increases almost linearly and reaches to 38 per cent when the longevity is 100. The effect of public pension is fairly sizable.

[Figure 2, 3, and 4 are here]

Figure 2 illustrates the optimal allocation of unskilled labor which is given by equations (29) and (31). At a starting point where the average longevity is 73.2, the labor share in the good sector is 70 per cent. In a process of population aging, it decreases gradually to the bottom of 45 per cent when the longevity is 100. Our model generates a fairly sizable reallocation of labor compared to Acemoglu and Guerrieri (2008).

Figure 3 illustrates the industry share in terms of value-added. When the average longevity is 73.2, we have $q_t y_{2t}/y_{1t} = \alpha/(1-\beta) = 0.375$. Thus, the share of good production sector is $y_{1t}/(y_{1t} + q_t y_{2t}) = 0.73$. The share decreases gradually to the bottom of 48 per cent when the longevity is 100. Our model suggests demographic changes and the related public policy account for the Petty-Clark's law in terms of both employment and value-added.

Figure 4 illustrates the optimal income components of old-age workers which is given by equations in Proposition 1 and 2. Without public pensions, the income share of labor is $\beta = 0.2$ and that of capital is $1 - \beta = 0.8$. The labor share is independent of population aging. In a process of population aging, however, public pension benefits crowd-out capital income. The pension share reaches to 50 per cent when the average longevity is 95.

4 Discussions

4.1 A decision on retirement

In the basic model, we assumed that every survivor works in the service sector. It seems restrictive because someone may well choose early retirement. In this subsection, we introduce an opportunity to retire into the basic model in a simplest way.

Suppose that a fraction $\sigma_t \in (0, 1]$ of individuals born at period t would like to work in the service sector if they survive into the second period of life and that a fraction $(1 - \sigma_t)$ of individuals would not.

The budget constraints for a candidate to be a old-age worker are

$$\begin{aligned} (1-\tau)w_t &= c_t + s_t \\ R_{t+1}s_t + \pi_{t+1} + (1-\eta)P_{t+1} &= q_{t+1}d_{t+1} \end{aligned}$$

where $0 \leq \eta \leq 1$ stands for a suspension rate for old-age workers.

Utility maximization gives

$$\begin{aligned} c_t^e &= \frac{I_t^e}{1+\gamma\theta} \\ d_{t+1}^e &= \frac{\gamma\theta}{1+\gamma\theta}\frac{R_{t+1}I_t^e}{q_{t+1}} \\ s_t^e &= \frac{\gamma\theta}{1+\gamma\theta}(1-\tau)w_t - \frac{1}{1+\gamma\theta}\frac{\pi_{t+1} + (1-\eta)P_{t+1}}{R_{t+1}} \end{aligned}$$

where I_t^e stands for lifetime full income,

$$I_t^e = (1-\tau)w_t + \frac{\pi_{t+1} + (1-\eta)P_{t+1}}{R_{t+1}}$$

A superscript e denotes the variables of old-age workers.

The budget constrains for a candidate to be a retiree are

$$\begin{aligned}(1-\tau)w_t &= c_t + s_t \\ R_{t+1}s_t + P_{t+1} &= q_{t+1}d_{t+1}\end{aligned}$$

Utility maximization gives

$$\begin{aligned}c_t^u &= \frac{I_t^u}{1+\gamma\theta} \\ d_{t+1}^u &= \frac{\gamma\theta}{1+\gamma\theta}\frac{R_{t+1}I_t^u}{q_{t+1}} \\ s_t^u &= \frac{\gamma\theta}{1+\gamma\theta}(1-\tau)w_t - \frac{1}{1+\gamma\theta}\frac{P_{t+1}}{R_{t+1}}\end{aligned}$$

where

$$I_t^u = (1-\tau)w_t + \frac{P_{t+1}}{R_{t+1}}$$

A superscript u denotes the variables of retirees.

If $I_t^e > I_t^u$, everyone would like to work. Otherwise everyone would like to retire. In equilibrium, the fraction σ_t is endogenously determined where $I_t^e = I_t^u$, i.e.,

$$\pi_{t+1} = \eta P_{t+1} \tag{33}$$

Aggregation gives

$$\begin{aligned}c_t &= \sigma_t c_t^e + (1-\sigma_t)c_t^u \\ &= \frac{1}{1+\gamma\theta}\left[(1-\tau)w_t + \frac{\sigma_t\pi_{t+1} + (1-\eta\sigma_t)P_{t+1}}{R_{t+1}}\right]\end{aligned}$$

$$\begin{aligned}d_{t+1} &= \sigma_t d_{t+1}^e + (1-\sigma_t)d_{t+1}^u \\ &= \frac{\gamma\theta}{1+\gamma\theta}\frac{R_{t+1}}{q_{t+1}}\left[(1-\tau)w_t + \frac{\sigma_t\pi_{t+1} + (1-\eta\sigma_t)P_{t+1}}{R_{t+1}}\right]\end{aligned}$$

$$\begin{aligned}s_t &= \sigma_t s_t^e + (1-\sigma_t)s_t^u \\ &= \frac{\gamma\theta}{1+\gamma\theta}(1-\tau)w_t - \frac{1}{1+\gamma\theta}\frac{\sigma_t\pi_{t+1} + (1-\eta\sigma_t)P_{t+1}}{R_{t+1}} \qquad (34)\end{aligned}$$

Conditions for the market of managers and the budget of public pension are modified as follows,

$$h_t = \theta\sigma_{t-1} \tag{35}$$

$$\tau w_t = (1-\eta\sigma_{t-1})\theta P_t \tag{36}$$

The allocation of unskilled labor is given by equations (22) and (23).

In the same way as the basic model, the present value of pension benefits and the present value of management income are respectively given by

$$\frac{P_{t+1}}{R_{t+1}} = \frac{\tau}{\alpha(1-\tau)}\frac{k_{t+1}}{1-\eta\sigma_t} \tag{37}$$

$$\frac{\pi_{t+1}}{R_{t+1}} = \frac{\beta}{1-\beta}\left[1 + \frac{\tau}{\alpha(1-\tau)}\right]\frac{k_{t+1}}{\sigma_t} \tag{38}$$

Substituting equations (37) and (38) into equation (34), and using equation (16), we have

$$\frac{k_{t+1}}{y_t} = \frac{\alpha\gamma\theta}{\alpha\gamma\theta + \frac{1}{1-\beta}\left(\alpha + \frac{\tau}{1-\tau}\right)}$$

which is the same as equation (28).

Finally, substituting equations (37) and (38) into equation (33), we have

$$\sigma_t = \frac{1}{\eta}\frac{\beta\alpha + \beta(1-\alpha)\tau}{\beta\alpha + (1-\beta\alpha)\tau} \tag{39}$$

for $\forall t$. Equation (39) shows the fraction of old-age workers is decreasing in τ and η, as expected. The pension scheme (τ, η) is restricted from the lower bound in order to ensure the interior condition $(0 < \sigma_t \leq 1)$, that is, the contribution rate and the suspension rate have to satisfy

$$\begin{aligned} \eta &> \underline{\eta} \equiv \frac{\beta(1-\alpha)}{1-\beta\alpha} \\ \tau &\geq \underline{\tau} \equiv \frac{(1-\eta)\beta\alpha}{\eta(1-\beta\alpha) - \beta(1-\alpha)} \end{aligned}$$

When $\alpha = 0.3$ and $\beta = 0.2$, we have $\underline{\eta} = 0.15$. Assuming that the suspension rate is 50 per cent, we have $\underline{\tau} = 0.09$. The opportunity of early retirement rises the threshold longevity that a public pension becomes effective.

[Figure 5 is here]

Figure 5 illustrates the rate of labor force participation of old-age workers when the contribution rate is set to be optimal (equation (31)). The suspension rate is set to be $\eta = 0.5$. The model works well when the average longevity exceeds 77.5 years old. Our model suggests the suspension rate should be lower when longevity is higher because a higher contribution rate induces more old-age workers to leave labor market.

4.2 A chance to work in the good sector

In the basic model we assumed that old-age workers are employed only in the service sector. It may be restrictive because some kind of management abilities would be useful in the good production sector. It can be shown that this generalization does not change the main result qualitatively.

Let us assume that the production functions in the good and the service sector are given by

$$\begin{aligned} y_{1t} &= f(k_t, h_{1t}, l_{1t}) = Ak_t^{\alpha_K} h_{1t}^{\alpha_H} l_{1t}^{1-\alpha_K-\alpha_H} \\ y_{2t} &= g(h_{2t}, l_{2t}) = Bh_{2t}^{\beta} l_{2t}^{1-\beta} \end{aligned}$$

where h_{1t} and h_{2t} stand for the employment of skilled labor in the good and the service sector, respectively. α_K and α_H are the income share of capital and skilled labor in the good sector.

The factor prices are given by

$$
\begin{aligned}
w_t &= (1-\alpha_K-\alpha_H)\frac{y_{1t}}{l_{1t}} = q_t(1-\beta)\frac{y_{2t}}{l_{2t}} \\
\pi_t &= \alpha_H\frac{y_{1t}}{h_{1t}} = q_t\beta\frac{y_{2t}}{h_{2t}} \\
1+r_t &= \alpha_K\frac{y_{1t}}{k_t}
\end{aligned}
$$

The industry ratio is given by

$$
IR_t = \frac{(1-\alpha_K-\alpha_H)l_{2t}}{(1-\beta)l_{1t}} = \frac{\alpha_H h_{2t}}{\beta h_{1t}} \tag{40}
$$

Equation (40) implies the allocation of skilled labor is determined by the allocation of unskilled labor in order to adjust the marginal productivity between the good sector and the service sector.

The market of skilled labor is modified as

$$
h_{1t}+h_{2t}=\theta \tag{41}
$$

In the same way as the basic model, the allocation of unskilled labor is given by

$$
l_{1t}=(1-\alpha_K-\alpha_H)(1-\tau) \tag{42}
$$

and $l_{2t}=1-l_{1t}$. Substituting equations (41) and (42) into equation (40), we have

$$
h_{1t}=\frac{\alpha_H\theta}{\alpha_H+\frac{\beta}{1-\beta}\left(\alpha_K+\alpha_H+\frac{\tau}{1-\tau}\right)} \tag{43}
$$

which is decreasing in τ. Public pensions reallocate skilled labor from the good sector to the service sector.

Finally, we derive the investment ratio as follows. The present value of pension benefits and that of management income are given by

$$
\begin{aligned}
\frac{P_{t+1}}{R_{t+1}} &= \tau\frac{1-\alpha_K-\alpha_H}{\alpha_K}\frac{k_{t+1}}{l_{1t+1}} \\
\frac{\pi_{t+1}}{R_{t+1}} &= \frac{\alpha_H}{\alpha_K}\frac{\theta}{h_{1t+1}}k_{t+1}
\end{aligned}
$$

Substituting them into the capital market clearing condition, and using equations (42) and (43), we have

$$
\frac{k_{t+1}}{y_{1t}}=\frac{\alpha_K\gamma\theta}{\alpha_K\gamma\theta+\frac{1}{1-\beta}\left(\alpha_K+\alpha_H+\frac{\tau}{1-\tau}\right)} \tag{44}
$$

Appendix B shows the optimal solutions are

$$
\left(\frac{k_{t+1}}{y_{1t}}\right)^* = \rho\alpha_K \tag{45}
$$

$$
l_1^* = \frac{1}{1+\frac{1-\beta}{1-\alpha_K-\alpha_H}IR^*} \tag{46}
$$

$$
h_1^* = \frac{\theta}{1+\frac{\beta}{\alpha_H}IR^*} \tag{47}
$$

where the industry ratio is given by

$$IR^* = \frac{(1-\rho\alpha_K)\gamma\theta}{\rho} \tag{48}$$

Comparing equations (44), (42), and (43) with equations (45), (46), and (47), respectively, we know there exists a unique contribution rate which achieves the optimal allocation. Specifically, we have the following proposition.

Proposition 4 *The optimal contribution rate is given by*

$$\tau^* = \frac{(1-\rho\alpha_K)(1-\beta)\gamma\theta - \rho(\alpha_K+\alpha_H)}{\rho(1-\alpha_H-\alpha_K)+(1-\alpha_K)(1-\beta)\gamma\theta} \tag{49}$$

which is increasing in θ. Public pensions effectively achieve the social optimum when $\theta \geq \tilde{\theta}$ where

$$\tilde{\theta} = \frac{\rho(\alpha_K+\alpha_H)}{(1-\rho\alpha_K)(1-\beta)\gamma} \tag{50}$$

$\tilde{\theta}$ is increasing in ρ, α_K, α_H, and β, and decreasing in γ.

5 Conclusions

In this paper we construct a theoretical model to examine whether and to what extent pay-as-you-go public pensions achieve a socially optimal allocation. In contrast to the existing literature we assume older people are economically active. They demand services more than younger people, and supply their labor resource in the emerging service sector. In a process of population aging, it is socially desirable to reallocate young-age unskilled labor from manufacturing to service sectors. We show that public pensions serve the purpose and that the positive role of public pensions increases with population aging.

The title of OECD (2006) cited in Introduction is "Live longer, work longer." The foreword reads as below. We hope our research gives an answer to at least one of the questions.

'Employment and social policies and practices that discourage work at an older age effectively deny older workers choice in when and how they retire. Moreover, in an era of rapid population ageing, they result in a waste of valuable resources that business, the economy and society can ill-afford. This has to stop. Policy reforms are needed to reverse the trend towards ever-earlier retirement. But what can governments do to bring about changes that are often unpopular with many voters? How can workers, employers and governments work together to guide our ageing society to a prosperous future?'

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Appendix A [Social optimum]
The social optimum is characterized by solving the following problem

$$\max \quad \sum_{t=0}^{\infty} \rho^t [u^y(c_t^y, d_t^y) + \gamma\theta u^o(c_{t+1}^o, d_{t+1}^o)]$$

subject to

$$\begin{aligned} y_{1t} &= c_t^y + \theta c_t^o + k_{t+1} && \text{(A1)} \\ y_{2t} &= d_t^y + \theta d_t^o && \text{(A2)} \\ l_{1t} + l_{2t} &= 1 && \text{(A3)} \\ h_t &= \theta \end{aligned}$$

where

$$\begin{aligned} u^y(c_t^y, d_t^y) &= (1-\mu)\ln c_t^y + \mu \ln d_t^y \\ u^o(c_{t+1}^o, d_{t+1}^o) &= (1-\lambda)\ln c_{t+1}^o + \lambda \ln d_{t+1}^o \\ y_{1t} &= Ak_t^{\alpha} l_{1t}^{1-\alpha} \\ y_{2t} &= Bh_t^{\beta} l_{2t}^{1-\beta} \end{aligned}$$

and $0 < \rho < 1$ stands for a social discount factor. μ and λ stand for the relative preference to service consumption when young and when old, respectively. We assume $\lambda \geq \mu$, i.e., the preference to service consumption increases with age. In a case where $\lambda = 1$ and $\mu = 0$, the structure is reduced to the basic model.

Let us set up the Lagrangian,

$$\begin{aligned} L_0 = & \sum_{t=0}^{\infty} \rho^t [u^y(c_t^y, d_t^y) + \gamma\theta u^o(c_{t+1}^o, d_{t+1}^o) + \pi_{1t}(y_{1t} - c_t^y - \theta c_t^o - k_{t+1}) \\ & + \pi_{2t}(y_{2t} - d_t^y - \theta d_t^o) + \pi_{3t}(1 - l_{1t} - l_{2t})] \end{aligned}$$

where π_{1t}, π_{2t}, and π_{3t} stand for multipliers attached to equations (A1), (A2), and (A3), respectively.

The first-order conditions require

$$\begin{aligned} \frac{\partial u^y}{\partial c_t^y} - \pi_{1t} &= 0 && \text{(A4)} \\ \frac{\partial u^y}{\partial d_t^y} - \pi_{2t} &= 0 && \text{(A5)} \\ \gamma\theta \frac{\partial u^o}{\partial c_{t+1}^o} - \rho\pi_{1t+1}\theta &= 0 && \text{(A6)} \\ \gamma\theta \frac{\partial u^o}{\partial d_{t+1}^o} - \rho\pi_{2t+1}\theta &= 0 && \text{(A7)} \\ -\pi_{1t} + \rho\pi_{1t+1}\frac{\partial y_{1t+1}}{\partial k_{t+1}} &= 0 && \text{(A8)} \\ -\pi_{3t} + \pi_{1t}\frac{\partial y_{1t}}{\partial l_{1t}} &= 0 && \text{(A9)} \\ -\pi_{3t} + \pi_{2t}\frac{\partial y_{2t}}{\partial l_{2t}} &= 0 && \text{(A10)} \end{aligned}$$

and the transversality condition is $\lim_{t\to\infty} \rho^t \pi_{1t} k_{t+1} = 0$.

From equations (A9) and (A10),

$$\begin{aligned} \pi_{3t} l_{1t} &= (1-\alpha)\pi_{1t} y_{1t} \quad &\text{(A11)} \\ \pi_{3t} l_{2t} &= (1-\beta)\pi_{2t} y_{2t} \quad &\text{(A12)} \end{aligned}$$

Substituting equations (A11) and (A12) into equation (A3), we have the optimal allocation of unskilled labor,

$$l_{1t}^* = \frac{1}{1 + \frac{1-\beta}{1-\alpha} IR_t} \tag{A13}$$

where IR_t stands for the industry ratio of the service sector to the good sector,

$$IR_t = \frac{\pi_{2t} y_{2t}}{\pi_{1t} y_{1t}}$$

Substituting equations (A5) and (A7) into equation (A2), we know, for $\forall t$,

$$\pi_{2t} y_{2t} = \mu + \lambda \frac{\gamma\theta}{\rho} \tag{A14}$$

Substituting equations (A4) and (A6) into equations (A1), we have

$$\pi_{1t} y_{1t} = 1 - \mu + (1-\lambda)\frac{\gamma\theta}{\rho} + \pi_{1t} k_{t+1} \tag{A15}$$

Equation (A8) gives

$$\pi_{1t} k_{t+1} = \rho\alpha \pi_{1t+1} y_{1t+1} \tag{A16}$$

From equations (A15) and (A16), we have a first-order difference equation of $\lambda_{1t} y_{1t}$,

$$\pi_{1t} y_{1t} = 1 - \mu + (1-\lambda)\frac{\gamma\theta}{\rho} + \rho\alpha \pi_{1t+1} y_{1t+1}$$

From the transversality condition, we know, for $\forall t$,

$$\pi_{1t} y_{1t} = \frac{1}{1-\rho\alpha}\left[1 - \mu + (1-\lambda)\frac{\gamma\theta}{\rho}\right] \tag{A17}$$

Equations (A14) and (A17) tell us the industry ratio is constant over time,

$$IR^* = \frac{(1-\rho\alpha)\left(\mu + \lambda\frac{\gamma\theta}{\rho}\right)}{1 - \mu + (1-\lambda)\frac{\gamma\theta}{\rho}} \tag{A18}$$

Substituting equation (A18) into equation (A13), we have

$$l_1^* = \frac{(1-\alpha)\left[1 - \mu + (1-\lambda)\frac{\gamma\theta}{\rho}\right]}{(1-\alpha)\left[1 - \mu + (1-\lambda)\frac{\gamma\theta}{\rho}\right] + (1-\beta)(1-\rho\alpha)\left(\mu + \lambda\frac{\gamma\theta}{\rho}\right)} \tag{A19}$$

Assuming that $\lambda = 1$ and $\mu = 0$, equation (A19) is equal to equation (29).

Equation (A14) gives the allocation of services,

$$\begin{aligned}\frac{d_t^y}{y_{2t}} &= \frac{\mu}{\mu+\lambda\frac{\gamma\theta}{\rho}}\\ \frac{\theta d_t^o}{y_{2t}} &= \frac{\lambda\frac{\gamma\theta}{\rho}}{\mu+\lambda\frac{\gamma\theta}{\rho}}\end{aligned}$$

Equation (A17) gives the allocation of goods,

$$\begin{aligned}\frac{c_t^y}{y_{1t}} &= \frac{(1-\rho\alpha)(1-\mu)}{1-\mu+(1-\lambda)\frac{\gamma\theta}{\rho}}\\ \frac{\theta c_t^o}{y_{1t}} &= \frac{(1-\rho\alpha)(1-\lambda)\frac{\gamma\theta}{\rho}}{1-\mu+(1-\lambda)\frac{\gamma\theta}{\rho}}\\ \frac{k_{t+1}}{y_{1t}} &= \rho\alpha \qquad \text{(A20)}\end{aligned}$$

Assuming $\lambda = 1$ and $\mu = 0$, we have $d_t^y = c_t^o = 0$, which corresponds to the basic model. Equation (A20) is equation (30) in the main body.

Let us denote the preference distance by

$$\varepsilon = \lambda - \mu \geq 0 \qquad \text{(A21)}$$

Eliminating μ in equation (A18) by equation (A20), we have

$$IR^* = \frac{(1-\rho\alpha)\left[\lambda\left(1+\frac{\gamma\theta}{\rho}\right)-\varepsilon\right]}{(1-\lambda)\left(1+\frac{\gamma\theta}{\rho}\right)+\varepsilon}$$

When $\varepsilon = 0$, i.e., the preference of individuals does not change, the industry ratio is independent of θ,

$$IR^* = \frac{(1-\rho\alpha)\lambda}{1-\lambda}$$

When $\varepsilon > 0$, i.e., the preference to service consumption increases with age, the industry ratio is increasing in θ.

Appendix B [Social optimum in Section 4.2]
The social optimum is characterized by solving the following problem

$$\max \quad \sum_{t=0}^{\infty} \rho^t (\ln c_t + \gamma\theta \ln d_{t+1})$$

subject to

$$\begin{aligned} y_{1t} &= c_t + k_{t+1} && \text{(B1)} \\ y_{2t} &= \theta d_t && \text{(B2)} \\ l_{1t} + l_{2t} &= 1 && \text{(B3)} \\ h_{1t} + h_{2t} &= \theta && \text{(B4)} \end{aligned}$$

where

$$\begin{aligned} y_{1t} &= A k_t^{\alpha_K} h_{1t}^{\alpha_H} l_{1t}^{1-\alpha_K-\alpha_H} \\ y_{2t} &= B h_{2t}^{\beta} l_{2t}^{1-\beta} \end{aligned}$$

Let us set up the Lagrangian,

$$\begin{aligned} L_0 = & \sum_{t=0}^{\infty} \rho^t [\ln c_t + \gamma\theta \ln d_{t+1} + \lambda_{1t}(y_{1t} - c_t - k_{t+1}) \\ & + \lambda_{2t}(y_{2t} - \theta d_t^o) + \lambda_{Lt}(1 - l_{1t} - l_{2t}) + \lambda_{Ht}(\theta - h_{1t} - h_{2t})] \end{aligned}$$

where λ_{1t}, λ_{2t}, λ_{Lt}, and λ_{Ht} stand for multipliers attached to equations (B1), (B2), (B3), and (B4), respectively.

The first-order conditions require

$$\begin{aligned} \frac{1}{c_t} - \lambda_{1t} &= 0 && \text{(B5)} \\ \frac{\gamma\theta}{d_{t+1}} - \rho\lambda_{2t+1}\theta &= 0 && \text{(B6)} \\ -\lambda_{1t} + \rho\lambda_{1t+1}\frac{\partial y_{1t+1}}{\partial k_{t+1}} &= 0 && \text{(B7)} \\ -\lambda_{Lt} + \lambda_{1t}\frac{\partial y_{1t}}{\partial l_{1t}} &= 0 && \text{(B8)} \\ -\lambda_{Lt} + \lambda_{2t}\frac{\partial y_{2t}}{\partial l_{2t}} &= 0 && \text{(B9)} \\ -\lambda_{Ht} + \lambda_{1t}\frac{\partial y_{1t}}{\partial h_{1t}} &= 0 && \text{(B10)} \\ -\lambda_{Ht} + \lambda_{2t}\frac{\partial y_{2t}}{\partial h_{2t}} &= 0 && \text{(B11)} \end{aligned}$$

and the transversality condition is $\lim_{t\to\infty} \rho^t \lambda_{1t} k_{t+1} = 0$.

From equations (B8) and (B9),

$$\begin{aligned} \lambda_{Lt} l_{1t} &= (1 - \alpha_K - \alpha_H)\lambda_{1t} y_{1t} \\ \lambda_{Lt} l_{2t} &= (1 - \beta)\lambda_{2t} y_{2t} \end{aligned}$$

Substituting them into equation (B3), we have the optimal allocation of unskilled labor,

$$l_{1t}^{*} = \frac{1}{1 + \frac{1-\beta}{1-\alpha_K-\alpha_H} IR_t} \tag{B12}$$

where IR_t stands for the industry ratio of the service sector to the good sector,

$$IR_t = \frac{\lambda_{2t} y_{2t}}{\lambda_{1t} y_{1t}} \tag{B13}$$

Equation (B12) is equation (46) in the main body.

From equations (B10) and (B11),

$$\begin{aligned} \lambda_{Ht} h_{1t} &= \alpha_H \lambda_{1t} y_{1t} \\ \lambda_{Ht} h_{2t} &= \beta \lambda_{2t} y_{2t} \end{aligned}$$

Substituting them into equation (B4), we have the optimal allocation of skilled labor,

$$h_{1t}^{*} = \frac{\theta}{1 + \frac{\beta}{\alpha_H} IR_t}$$

which is equation (47) in the main body.

From equations (B2) and (B6), we have

$$\lambda_{2t} y_{2t} = \frac{\gamma\theta}{\rho} \tag{B14}$$

Substituting equations (B5) and (B7) into (B1), we have

$$\lambda_{1t} y_{1t} = 1 + \rho\alpha_K \lambda_{1t+1} y_{1t+1}$$

Since $0 < \rho\alpha_K < 1$, the transversality condition requires, for $\forall t$,

$$\lambda_{1t} y_{1t} = \frac{1}{1-\rho\alpha_K} \tag{B15}$$

From equation (B15), we have

$$\frac{k_{t+1}}{y_t} = \rho\alpha_K$$

which is equation (45) in the main body.

Finally, substituting equations (B14) and (B15) into equation (B13), we have

$$IR_t = \frac{(1-\rho\alpha_K)\gamma\theta}{\rho}$$

which is equation (48) in the main body.

Figure 1. Optimal contribution rate

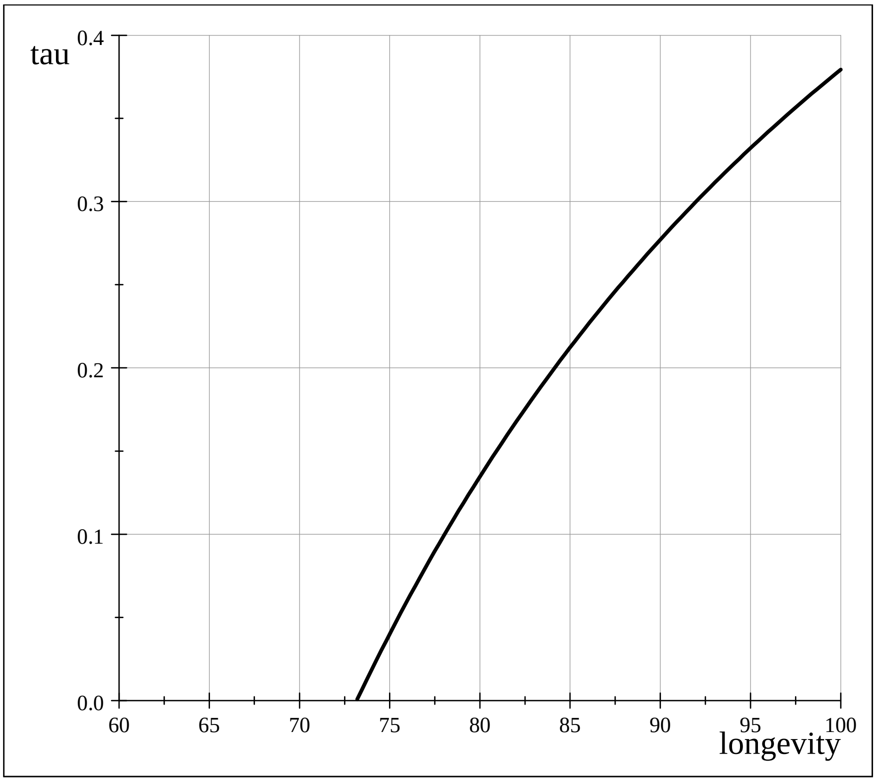
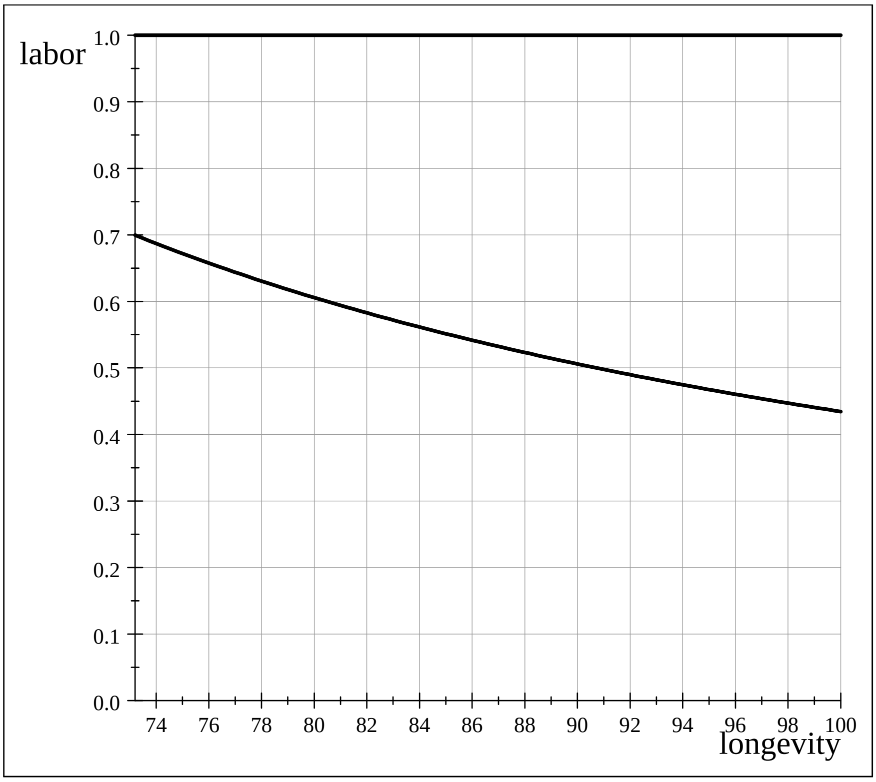
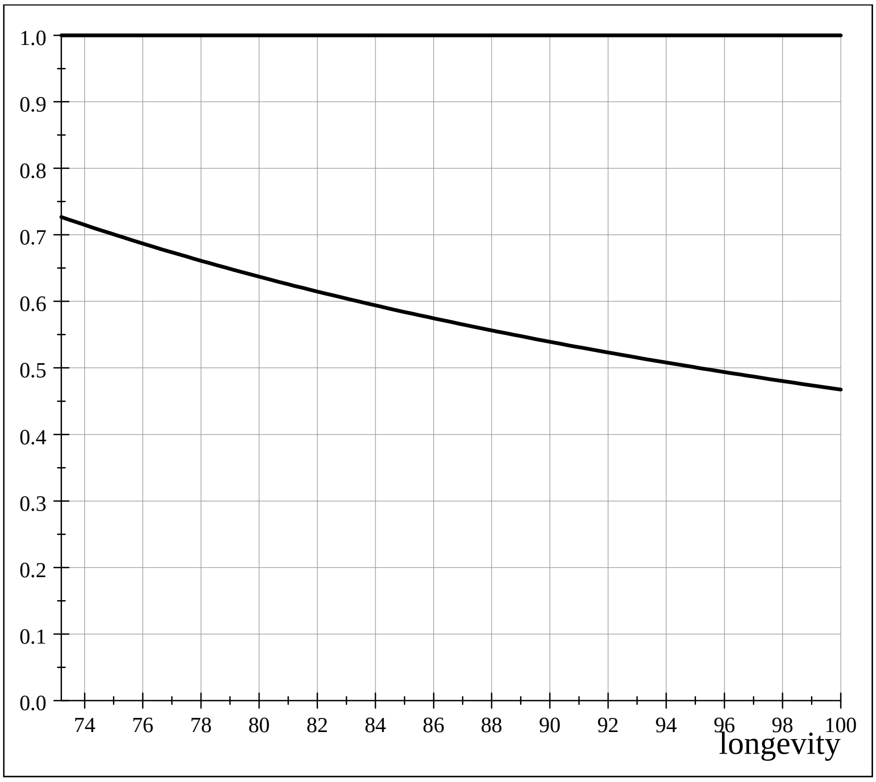


Figure 2. Industry share, Unskilled labor



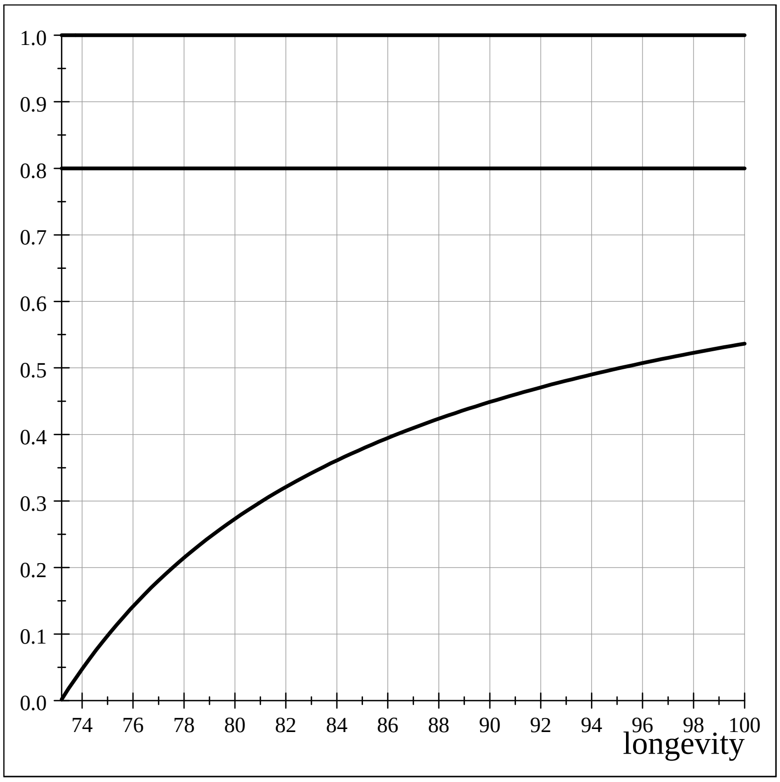
The employment of young-age workers moves from the good to the service sector.

Figure 3. Industry share, Value-added



The share of value-added in the good production sector decreases with population aging.

Figure 4. Income components of old-age workers



The share of labor income, capital income, and pension benefits from top to bottom.

Figure 5. Participation rate of old-age workers